ETH zürich

Network Security Group
Department of Computer Science

Model-Based Insights on the Performance, Fairness, and Stability of BBR

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ETH Zurich

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TU Berlin & Fraunhofer SIT

IMC 2022, Nice

The journey of BBR development

2016:
First version presented by Google

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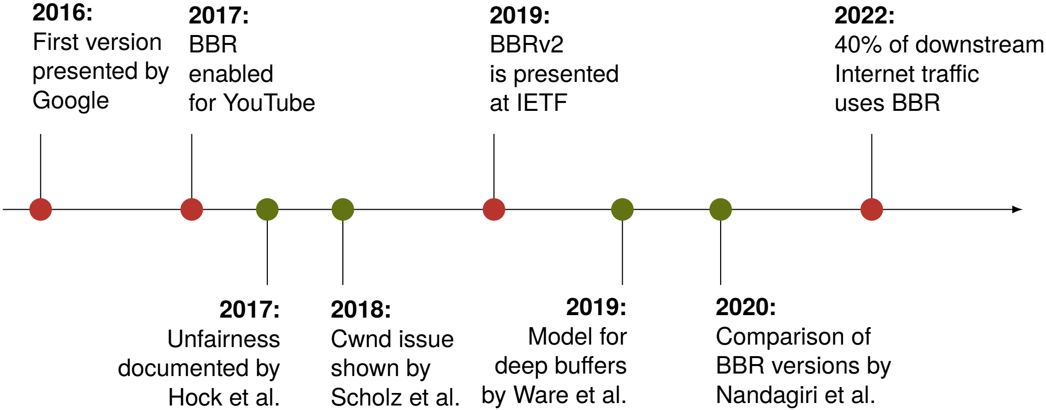
2016:
First version presented by Google

2017:
BBR enabled for YouTube

2019:
BBRv2 is presented at IETF

2022:
40% of downstream Internet traffic uses BBR

The journey of BBR development



Previous research on BBR leaves open important questions

The approaches of prior research have limitations:

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Steady-state models

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Steady-state models

No expression of transient effects
Transient phenomena (e.g., convergence behavior) are ignored, although highly relevant

A fluid model can fill the gaps in BBR analysis

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Fluid models enable:

Expression of transient effects
Fluid models allow to investigate if/how the CCA converges to an equilibrium (stability analysis)

Efficient simulation
Differential equations can be efficiently solved for a wide range of scenarios
(e.g., simulation cost is independent of flow rates)

Our contribution: A BBR analysis based on a fluid model

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Fluid-model design

Formalization of BBR behavior

Design of new techniques

$$\dot{\tau}_i^{\min} = -\Gamma \quad \tau_i^{\min}(t) - \tau_i(t - d_i^{\mathrm{p}})$$

$$\dot{x}_i^{\mathrm{btl}} = \sigma\left(t_i^{\mathrm{pbw}} - T_i^{\mathrm{pbw}} + 0.01\right) \cdot \left(x_i^{\max} - x_i^{\mathrm{btl}}\right)$$

$$x_i^{\mathrm{dlv}} = \frac{x_i\left(t - d_i^{\mathrm{p}}\right)}{y_\ell \quad t - d_{i,\ell}^{\mathrm{b}}} \cdot \begin{cases} C_\ell & \text{if } q_\ell(t - d_{i,\ell}^{\mathrm{b}}) > 0 \\ y_\ell\left(t - d_{i,\ell}^{\mathrm{b}}\right) & \text{otherwise} \end{cases}$$

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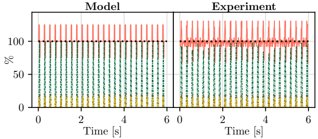
Experimental validation

Confirmation of prior insights

Generation of new insights



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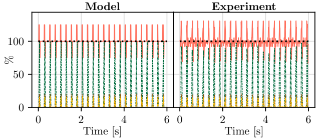
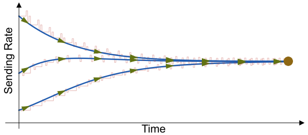


Theoretical stability analysis

Characterization of equilibria

Proof of asymptotic stability



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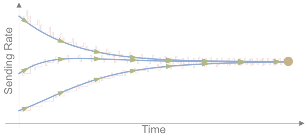

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Reno control loop: Congestion-window size w

if ack_received **then**
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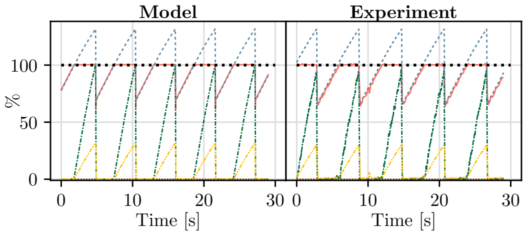

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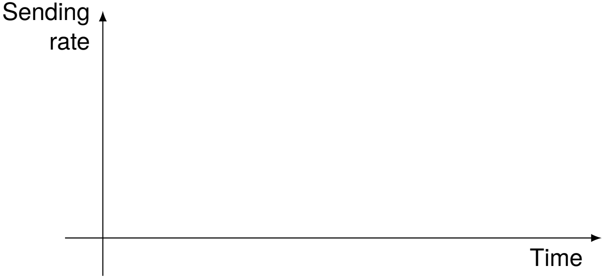
BBRv1 bandwidth probing:



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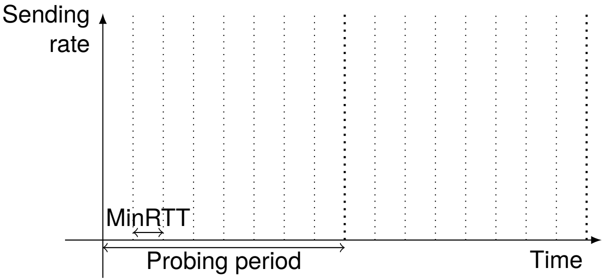
Probing periods of 8 MinRTT (phases)



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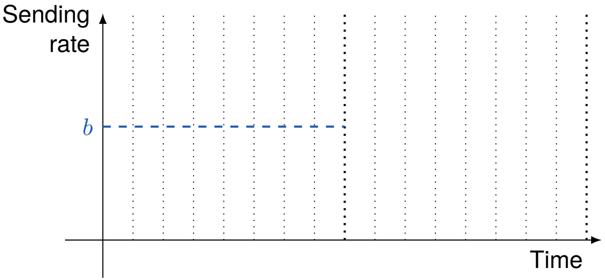

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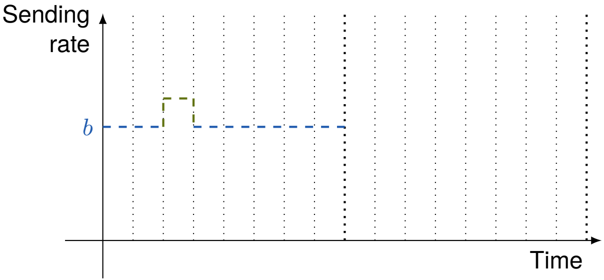

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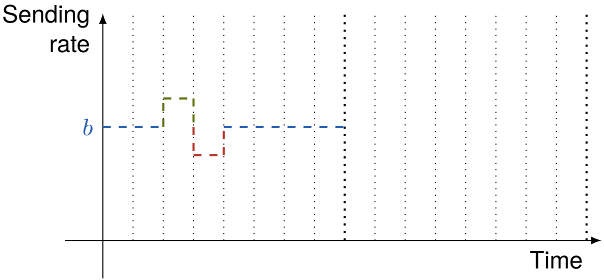

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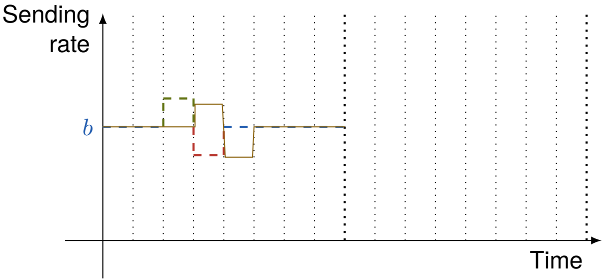

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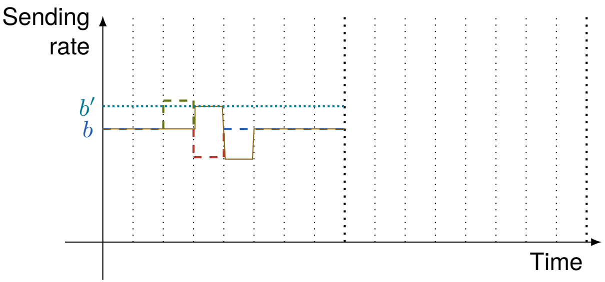
Maximum delivery rate is new bottleneck estimate b'



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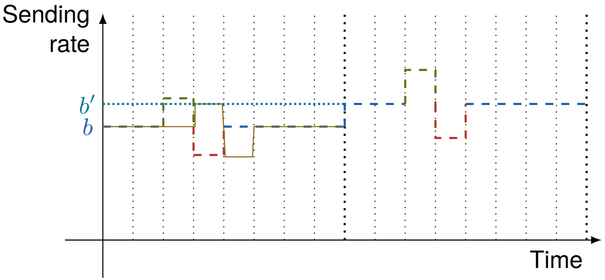

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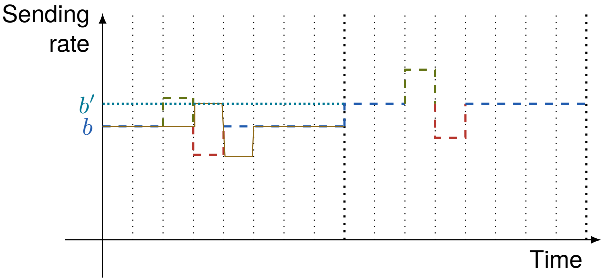

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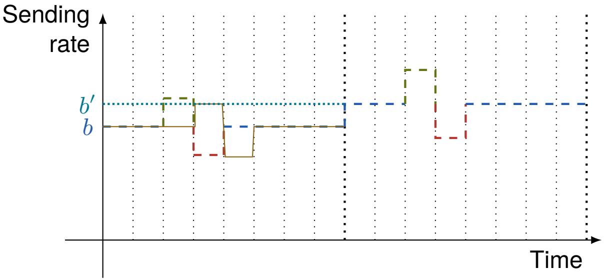

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Probing pulses? Maximum tracking? Periodic adjustment? Random phases?

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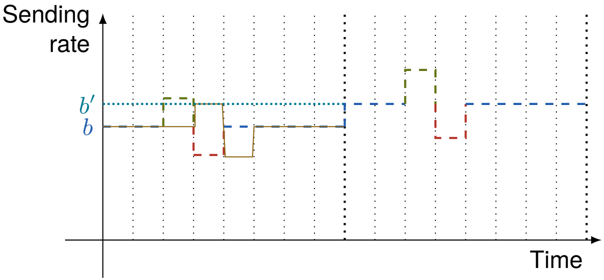

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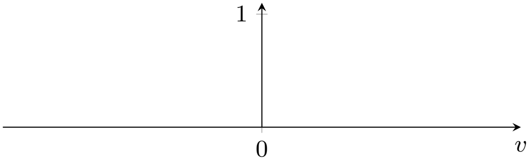
$$\sigma(v) = \frac{1}{1 + e^{-K \cdot v}}$$



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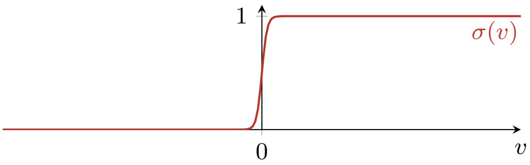

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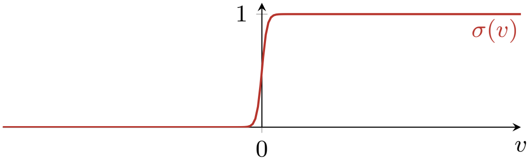

Pulse function

$$\Phi(t, \phi) = \sigma\left(t - \phi \cdot \tau_{\min}\right) \cdot \sigma\left((\phi + 1) \cdot \tau_{\min} - t\right)$$

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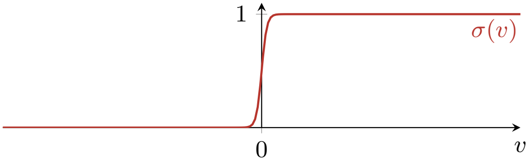

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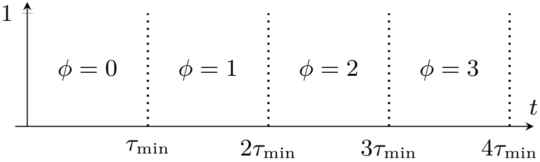

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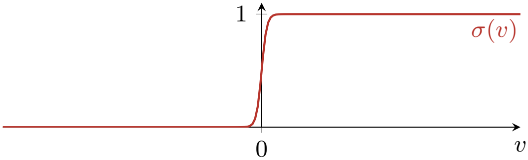

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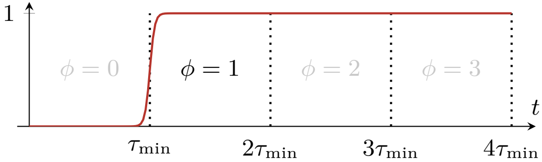

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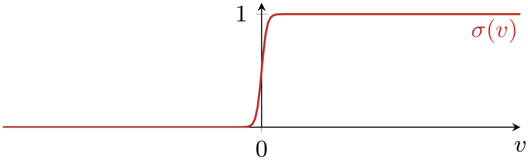
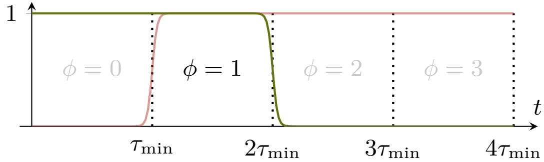


Pulse function

$$\Phi(t, \phi) = \sigma\left(t - \phi \cdot \tau_{\text{min}}\right) \cdot \sigma\left((\phi + 1) \cdot \tau_{\text{min}} - t\right)$$



Representing BBR in a fluid model: Probing pulses

Sigmoid function

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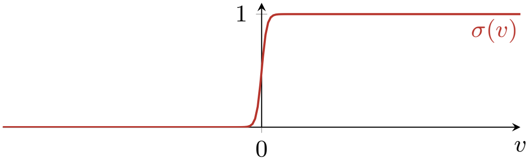

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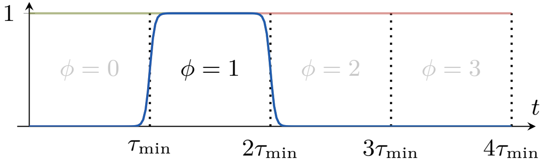

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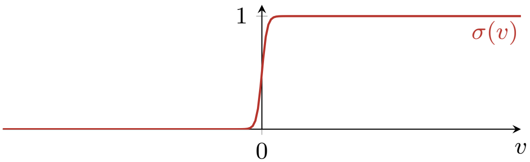

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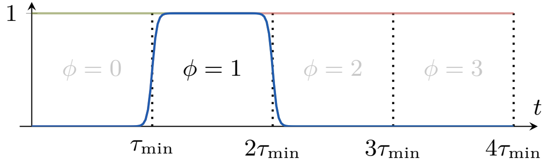
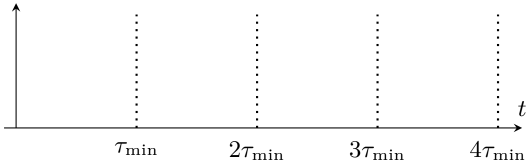


Pacing rate

$$x^{\text{pcg}}(t) = x^{\text{btl}}(t) \cdot \left(1 + {}^{1}\!/_{4}\Phi(t, \phi') - {}^{1}\!/_{4}\Phi(t, \phi' + 1)\right)$$



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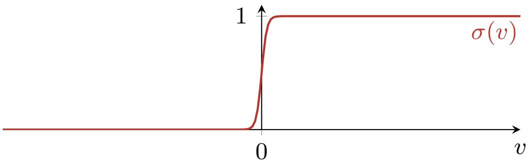

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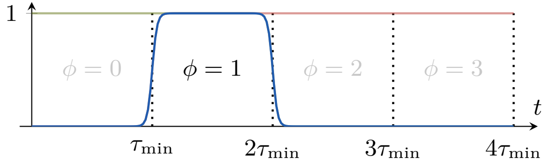

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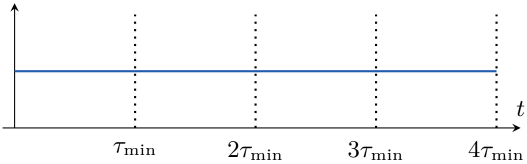

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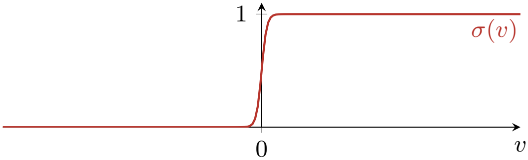

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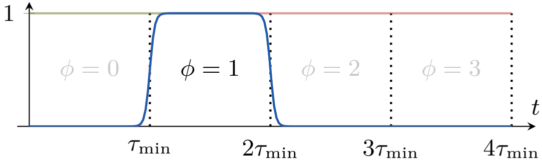

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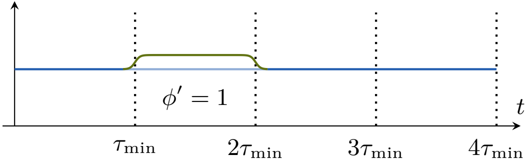

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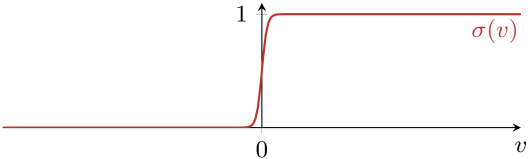

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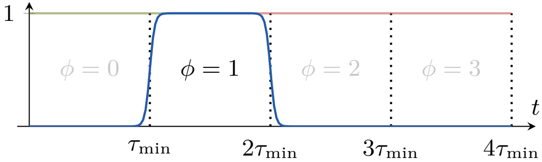

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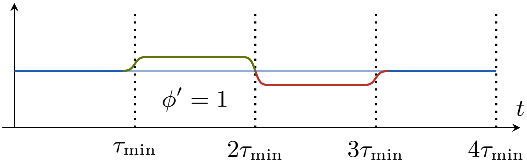

Representing BBR in a fluid model: Maximum tracking

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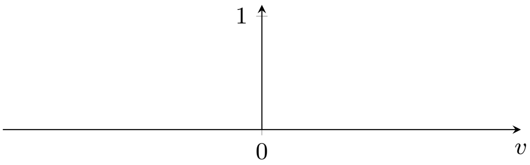

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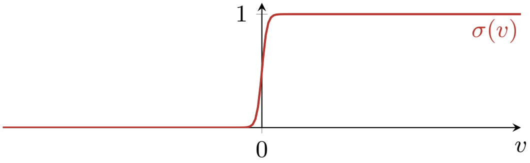


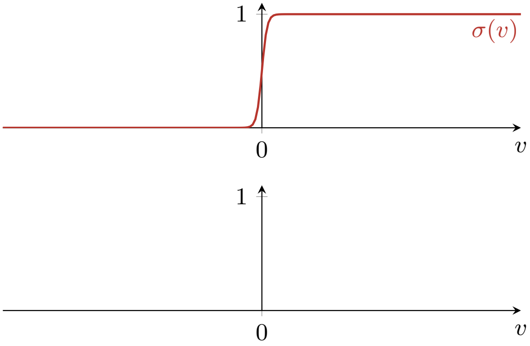
Representing BBR in a fluid model: Maximum tracking

Sigmoid function

$$\sigma(v) = \frac{1}{1 + e^{-K \cdot v}}$$

Maximum function

$$\Gamma(v) = v \cdot \sigma(v)$$



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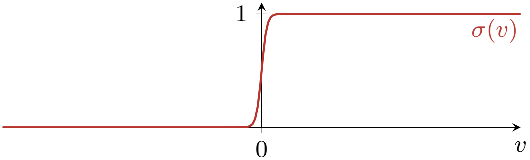

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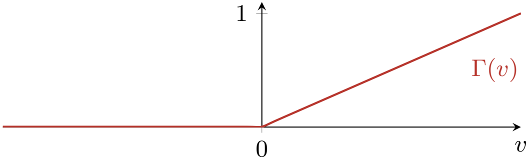

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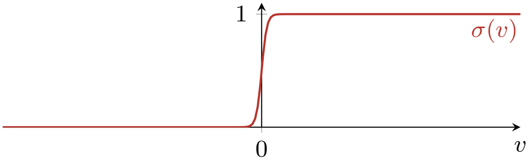

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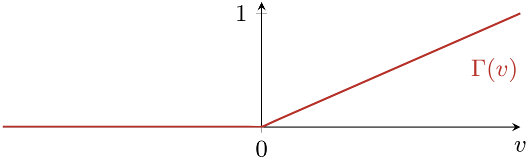

Tracking of maximum delivery rate

$$\dot{x}^{\max}(t) = \Gamma\left(x^{\mathrm{dlv}}(t) - x^{\max}(t)\right)$$

Representing BBR in a fluid model: Maximum tracking

Sigmoid function

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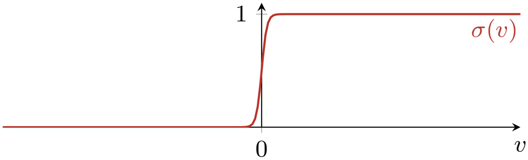

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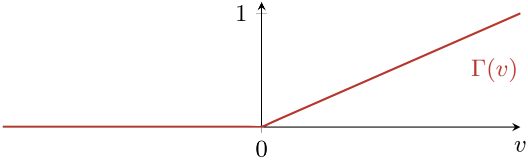

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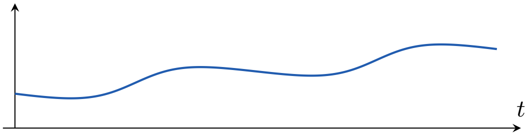

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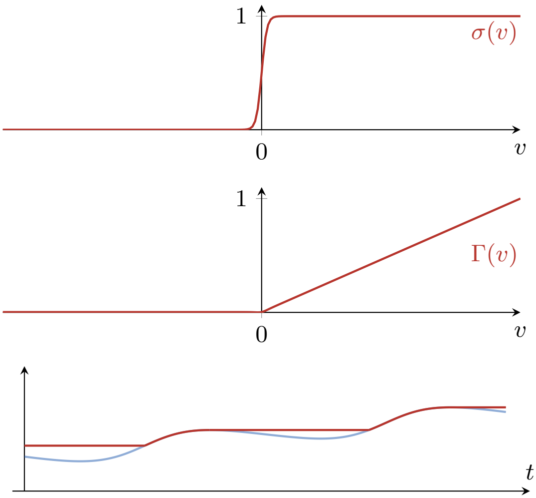


Representing BBR in a fluid model: End result

Network model

$$y_\ell = \sum_{i \in U_\ell} x_i\left(t - d^{\mathrm{f}}_{i,\ell}\right),$$

$$\dot{q}_\ell = (1 - p_\ell) \cdot y_\ell - C_\ell, \quad q_\ell(t) \in [0, B_\ell],$$

$$\tau_{\pi_i} = \sum_{\ell \in \pi_i} \tau_\ell = \sum_{\ell \in \pi_i} d_\ell + \frac{q_\ell}{C_\ell}.$$

$$p_\ell(t) = \sigma\left(y_\ell(t) - C_\ell\right) \cdot \left(1 - \frac{C_\ell}{y_\ell}\right) \cdot \left(\frac{q_\ell}{B_\ell}\right)^L$$

$$p_\ell = \frac{q_\ell}{B_\ell} \quad \in [0, 1].$$

$$p_{\pi_i}(t) = 1 - \prod_{\ell \in \pi}\left(1 - p_\ell\left(t + d^{\mathrm{f}}_{i,\ell}\right)\right) \approx \sum_{\ell \in \pi} p_\ell\left(t + d^{\mathrm{f}}_{i,\ell}\right).$$

$$x_i = \frac{w_i}{\tau_i}.$$

Basic BBR model

$$\dot{\tau}^{\min}_i = -\Gamma\left(\tau^{\min}_i(t) - \tau_i(t - d^{\mathrm{p}}_i)\right)$$

$$\Gamma(v) = v \cdot \sigma(v).$$

$$\Delta m^{\mathrm{prt}}_i = \sigma\left(t^{\mathrm{prt}}_i - T^{\mathrm{prt}}_i\right) \cdot \left((1 - m^{\mathrm{prt}}_i) - m^{\mathrm{prt}}_i\right)$$

$$T^{\mathrm{prt}}_i = m^{\mathrm{prt}}_i \cdot 0.2 + \left(1 - m^{\mathrm{prt}}_i\right) \cdot 10$$

$$\dot{t}^{\mathrm{prt}}_i = 1 - \sigma\left(t^{\mathrm{prt}}_i - T^{\mathrm{prt}}_i\right) \cdot t^{\mathrm{prt}}_i - \sigma\left(\tau^{\min}_i - \tau_i(t - d_\ell)\right) \cdot t^{\mathrm{prt}}_i$$

$$x_i = m^{\mathrm{prt}}_i \cdot \frac{w^{\mathrm{prt}}_i}{\tau_i} - \left(1 - m^{\mathrm{prt}}_i\right) \cdot x^{\mathrm{pbw}}_i$$

$$x^{\mathrm{pbw}}_i = \min\left(\frac{w^{\mathrm{pbw}}_i}{\tau_i}, x^{\mathrm{pcg}}_i\right)$$

$$\dot{t}^{\mathrm{pbw}}_i = 1 - \sigma\left(t^{\mathrm{pbw}}_i - T^{\mathrm{pbw}}_i\right) \cdot t^{\mathrm{pbw}}_i$$

$$x^{\mathrm{dlv}}_i = \frac{x_i(t - d^{\mathrm{p}}_i)}{y_\ell\left(t - d^{\mathrm{b}}_{i,\ell}\right)} \cdot \begin{cases} C_\ell & \text{if } q_\ell(t - d^{\mathrm{b}}_{i,\ell}) > 0 \\ y_\ell\left(t - d^{\mathrm{b}}_{i,\ell}\right) & \text{otherwise} \end{cases}$$

$$\dot{x}^{\max}_i = \Gamma(x^{\mathrm{dlv}}_i - x^{\max}_i) - \sigma(0.01 - t^{\mathrm{pbw}}_i) \cdot x^{\max}_i$$

$$\dot{v}_i = x_i - x^{\mathrm{dlv}}_i$$

BBRv1 model

$$\dot{x}^{\mathrm{btl}}_i = \sigma\left(t^{\mathrm{pbw}}_i - T^{\mathrm{pbw}}_i + 0.01\right) \cdot \left(x^{\max}_i - x^{\mathrm{btl}}_i\right)$$

$$\Phi_i(t, \phi) = \sigma\left(t^{\mathrm{pbw}}(t) - \phi \cdot \tau^{\min}_i\right) \cdot \sigma\left((\phi + 1) \cdot \tau^{\min}_i - t^{\mathrm{pbw}}\right)$$

$$x^{\mathrm{pcg}}_i = x^{\mathrm{btl}}_i \cdot \left(1 + \tfrac{1}{4} \cdot \Phi_i(t, \phi_i) - \tfrac{1}{4} \cdot \Phi_i(t, \phi_i + 1)\right)$$

$$w^{\mathrm{prt}}_i = 4$$

$$w^{\mathrm{pbw}}_i = 2 \cdot \overline{w}_i = 2 \cdot x^{\mathrm{btl}}_i \cdot \tau^{\min}_i$$

BBRv2 model

$$T^{\mathrm{pbw}}_i = \min\left(62 \cdot \tau^{\min}_i, 2 + \frac{i}{N}\right)$$

$$x^{\mathrm{pcg}}_i = x^{\mathrm{btl}}_i \cdot \left(1 + \tfrac{1}{4} \cdot \sigma\left(t^{\mathrm{pbw}}_i - \tau^{\min}_i\right) \cdot \left(1 - m^{\mathrm{dwn}}_i\right) - \tfrac{1}{4} \cdot m^{\mathrm{dwn}}_i\right)$$

$$\begin{aligned} \Delta m^{\mathrm{dwn}}_i = \; & (1 - m^{\mathrm{crs}}_i) \cdot (1 - m^{\mathrm{dwn}}_i) \cdot \sigma \;\; t^{\mathrm{pbw}}_i - \tau^{\min}_i \\ & \cdot \min\left(\sigma\left(v_i - 5/4 \cdot \overline{w}_i\right) + \sigma\left(p_{\pi i} - 0.02\right), \; 1\right) \\ & - m^{\mathrm{dwn}}_i \cdot \sigma\left(w^-_i - v_i\right) \end{aligned}$$

$$\Delta m^{\mathrm{crs}}_i = -\Delta m^{\mathrm{dwn}}_i - \sigma\left(t^{\mathrm{pbw}}_i - T^{\mathrm{pbw}}_i\right) \cdot m^{\mathrm{crs}}_i$$

$$\dot{x}^{\mathrm{btl}}_i = m^{\mathrm{dwn}}_i \cdot \left(\max\left(x^{\max}_i, \; x^{\max}_i(t - T^{\mathrm{pbw}})\right) - x^{\mathrm{btl}}_i\right)$$

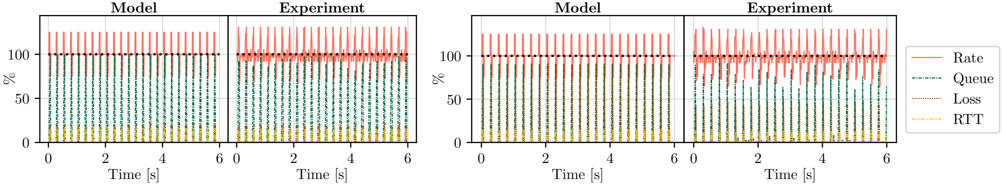
$$\begin{aligned} \dot{w}^{\mathrm{hi}}_i = \; & (1 - m^{\mathrm{crs}}_i) \cdot \sigma\left(t^{\mathrm{pbw}}_i - \tau^{\min}_i\right) \cdot \sigma\left(v_i - w^{\mathrm{hi}}\right) \cdot 2^{t^{\mathrm{pbw}}_i / \tau^{\min}_i} \\ & - \sigma\left(p_{\pi_i} - 0.02\right) \cdot \frac{0.3}{\tau^{\min}_i} \cdot w^{\mathrm{hi}}_i \end{aligned}$$

$$\dot{w}^{\mathrm{lo}}_i = (1 - m^{\mathrm{crs}}_i) \cdot \left(w^-_i - w^{\mathrm{lo}}_i\right) - m^{\mathrm{crs}}_i \cdot \sigma(p_{\pi_i}) \cdot \frac{0.3 w^{\mathrm{lo}}_i}{\tau^{\min}_i} \cdot$$

$$w^{\mathrm{pbw}}_i = \min\left(2 \cdot \overline{w}_i, \; (1 - m^{\mathrm{crs}}_i) \cdot w^{\mathrm{hi}}_i + m^{\mathrm{crs}}_i \cdot w^{\mathrm{lo}}_i\right)$$

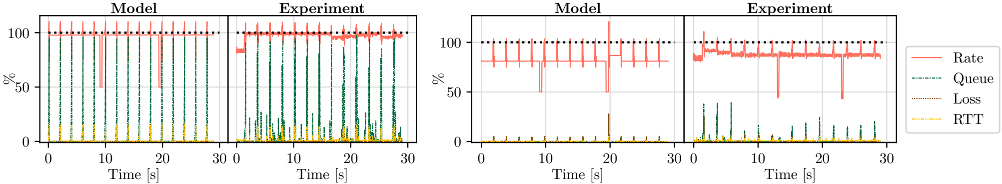
$$w^{\mathrm{prt}}_i = \frac{\overline{w}_i}{2}$$

Representing BBR in a fluid model: End result



(a) Drop-tail (b) RED

BBRv1.



(a) Drop-tail (b) RED

BBRv2.

Our contribution: A BBR analysis based on a fluid model

Fluid-model design

Formalization of BBR behavior

Design of new techniques

$$\dot{\tau}_i^{\min} = -\Gamma\left(\tau_i^{\min}(t) - \tau_i(t - d_i^{\mathrm{p}})\right)$$

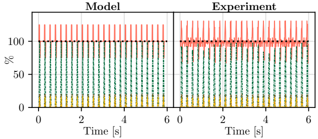
$$\dot{x}_i^{\mathrm{btl}} = \sigma\left(t_i^{\mathrm{pbw}} - T_i^{\mathrm{pbw}} + 0.01\right) \cdot \left(x_i^{\max} - x_i^{\mathrm{btl}}\right)$$

$$x_i^{\mathrm{dlv}} = \frac{x_i(t - d_i^{\mathrm{p}})}{y_\ell\left(t - d_{i,\ell}^{\mathrm{b}}\right)} \cdot \begin{cases} C_\ell & \text{if } q_\ell(t - d_{i,\ell}^{\mathrm{b}}) > 0 \\ y_\ell\left(t - d_{i,\ell}^{\mathrm{b}}\right) & \text{otherwise} \end{cases}$$

Experimental validation

Confirmation of prior insights

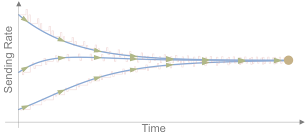
Generation of new insights



Theoretical stability analysis

Characterization of equilibria

Proof of asymptotic stability



Experimental validation of BBR fluid model

Configuration

Topology
Dumbbell topology Single bottleneck

Congestion-control algorithms
Homogeneous or heterogeneous (balanced)

Experimental validation of BBR fluid model

Configuration

Topology

Dumbbell topology
Single bottleneck

Congestion-control algorithms

Homogeneous or heterogeneous (balanced)

Evaluation tools

Fluid-model simulator

Solution of differential equations (Method of steps)

Experiment environment

Emulation with Mininet
Load generation with iperf

Experimental validation of BBR fluid model

Configuration

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Dumbbell topology
Single bottleneck

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Homogeneous or heterogeneous (balanced)

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Experiment environment

Emulation with Mininet
Load generation with iperf

Result validation

Trace validation

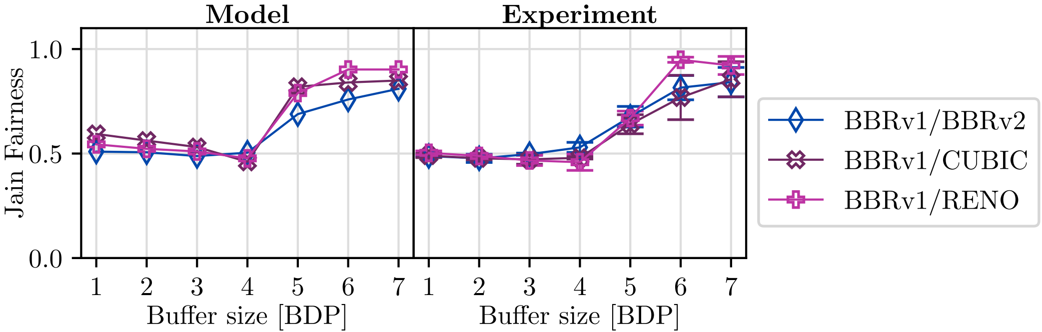
Evolution of network metrics over time for single flow

Aggregate-result validation

Network metrics (aggregated over time) for multiple flows

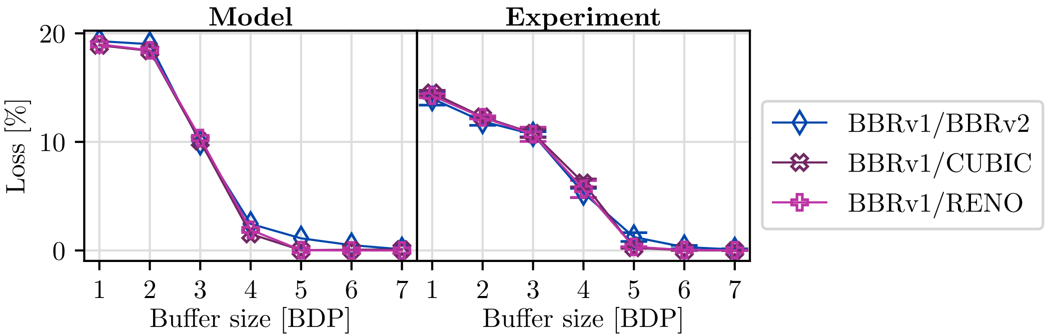
Confirmation of prior insights: Unfairness of BBRv1

Previous insight: BBRv1 is unfair towards loss-sensitive CCAs in shallow buffers.



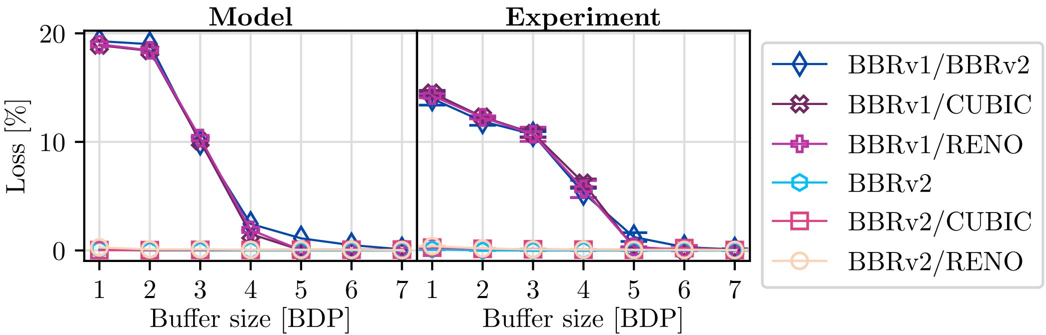
Confirmation of prior insights: High loss of BBRv1

Previous insight: BBRv1 leads to high loss in shallow buffers.



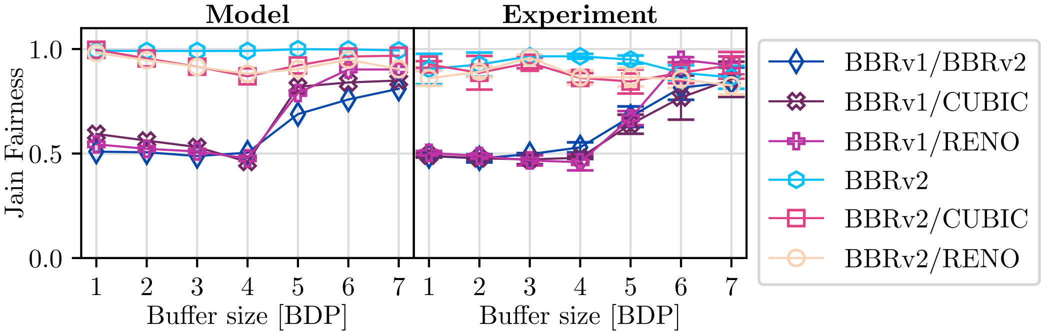
Confirmation of prior insights: Improved loss in BBRv2

Previous insight: BBRv2 leads to little loss (comparable to loss-based CCAs).



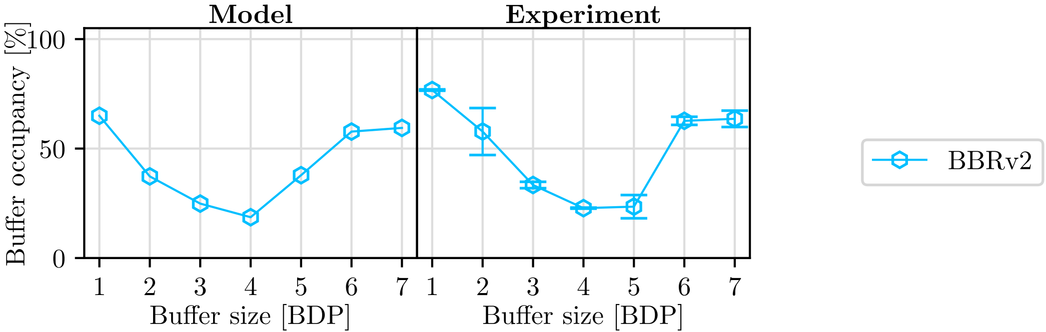
Confirmation of prior insights: Improved fairness in BBRv2

Previous insight: BBRv2 is quite fair to loss-based CCAs (under a drop-tail queuing discipline).



Generation of new insights: Bufferbloat in BBRv2

New insight: BBRv2 leads to intense queuing in large buffers.



Generation of new insights: Bufferbloat in BBRv2

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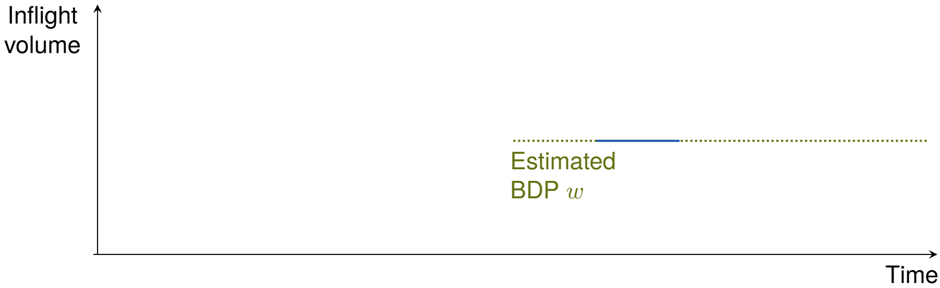
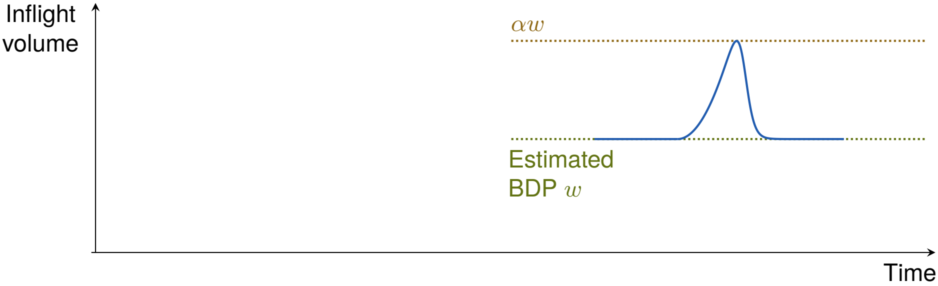


Generation of new insights: Bufferbloat in BBRv2



Generation of new insights: Bufferbloat in BBRv2

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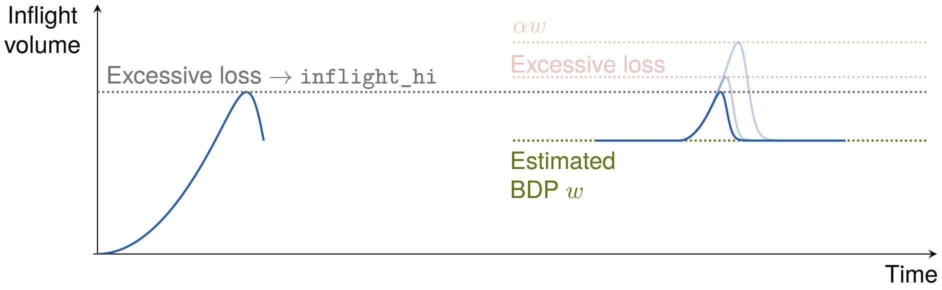
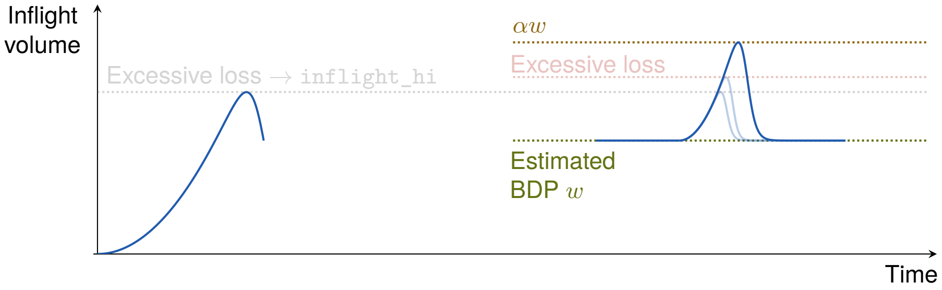


Generation of new insights: Bufferbloat in BBRv2

Large buffers disable loss-based safeguards

Generation of new insights: Bufferbloat in BBRv2



Large buffers disable loss-based safeguards

$\Longrightarrow$ More aggressive probing $\Longrightarrow$ Higher delivery rate
$\Longrightarrow$ Higher estimated BDP $\Longrightarrow$ Higher buffer utilization

Generation of new insights: Bufferbloat in BBRv2

Large buffers disable loss-based safeguards

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Our fluid model reproduces this dynamic effect

Our contribution: A BBR analysis based on a fluid model

Fluid-model design

Formalization of BBR behavior

Design of new techniques

$$\dot{\tau}_i^{\min} = -\Gamma\left(\tau_i^{\min}(t) - \tau_i(t - d_i^{\mathrm{p}})\right)$$

$$\dot{x}_i^{\mathrm{btl}} = \sigma\left(t_i^{\mathrm{pbw}} - T_i^{\mathrm{pbw}} + 0.01\right) \cdot \left(x_i^{\max} - x_i^{\mathrm{btl}}\right)$$

$$x_i^{\mathrm{dlv}} = \frac{x_i(t - d_i^{\mathrm{p}})}{y_\ell\left(t - d_{i,\ell}^{\mathrm{b}}\right)} \cdot \begin{cases} C_\ell & \text{if } q_\ell(t - d_{i,\ell}^{\mathrm{b}}) > 0 \\ y_\ell\left(t - d_{i,\ell}^{\mathrm{b}}\right) & \text{otherwise} \end{cases}$$

Experimental validation

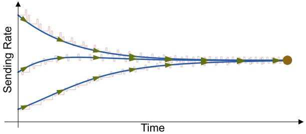
Confirmation of prior insights

Generation of new insights

Theoretical stability analysis

Characterization of equilibria

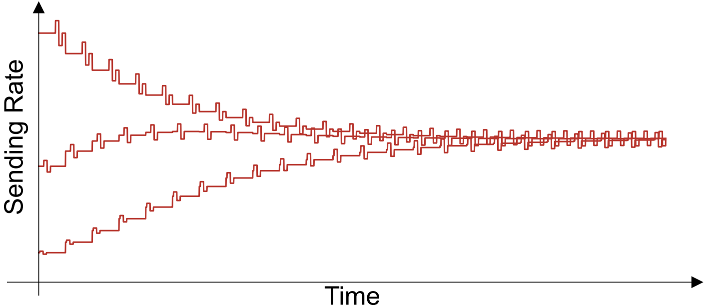
Proof of asymptotic stability



Theoretical stability analysis: Overview

Fluid model

Full fluid model
(used for simulation)



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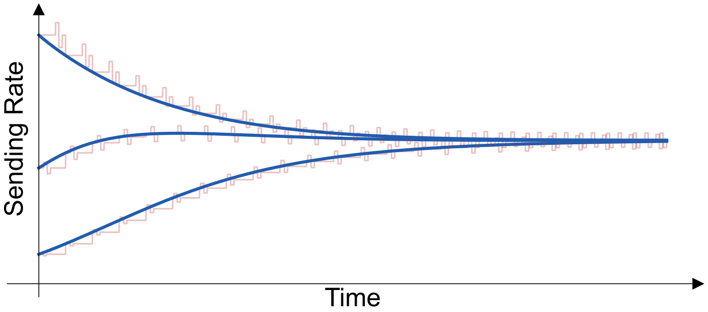
Reduced fluid model

High-level model
(macroscopic behavior)



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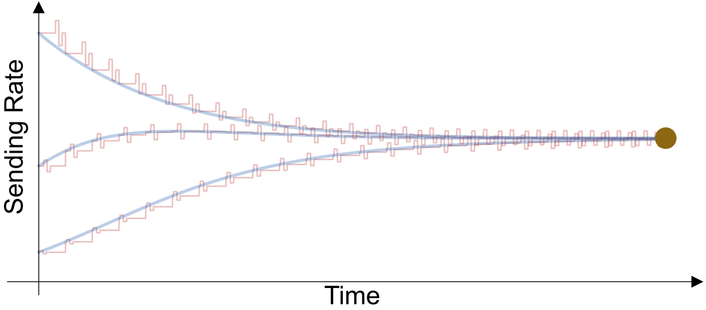
Equilibria

Rate distribution &
queue length in steady state



Theoretical stability analysis: Overview

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High-level model
(macroscopic behavior)

Equilibria

Rate distribution &
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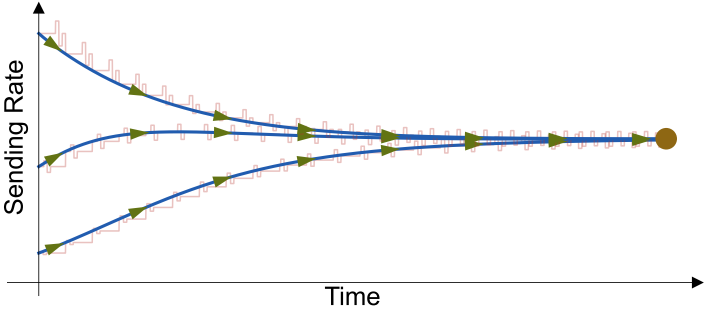
Asymptotic stability

Proof of attractiveness
(Lyapunov method)



Proof result: BBRv1 and BBRv2 converge to equilibria.
Equilibria may be unfair!

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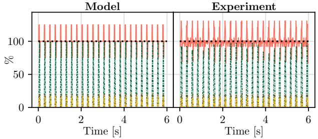

Theoretical stability analysis

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